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Short-time critical behaviour of anisotropic cubic systems with long-range interaction

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Abstract

The renormalization group approach is applied to the study of the short-time critical behaviour of the d-dimensional n-component anisotropic cubic spin systems with long-range interaction of the form $p^{\sigma} s_p s_{-p}$ in momentum space. Firstly, the system is quenched from a high temperature to the critical temperature and then relaxes to equilibrium within the model A dynamics. The asymptotic scaling laws and the initial slip exponents θ' and θ of the order parameter and the response function, respectively, are calculated to the second order in $\epsilon = 2\sigma - d$. For $1 \leqslant d < 2\sigma$ and $n > n_c$, the cubic anisotropy affects the short-time critical behaviour.

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1. Introduction

In recent years, much attention has been paid to the short-time critical dynamics [1–7]. The short-time phenomena arise at times just after a microscopic time scale t_{mic} needed by the system to remember only the macroscopic condition and to forget all specific microscopic details. It is a critical relaxation in a regime far from the equilibrium state. The corresponding time regime is also called critical initial slip in order to distinguish it from the uninteresting microscopic time interval between zero and t_{mic}. Since the pioneering analytical study of [1], universal short-time scalings have been found in various models [4, 6, 7]. When the system is quenched from a high temperature T_i to a heat bath at the critical temperature $T_c \ll T_i$, in the short-time regime not only does the order parameter show a power-law increase $m(t) \sim t^{\theta'}$ with a new exponent θ', but also the response function has $G_p(t, t') \sim (t/t')^{\theta}$ for $t' \to 0$. For a short time after quenching the scaling behaviour is governed by the initial slip exponents θ and θ'.

The short-time dynamics has been thoroughly investigated for models with short-range interaction (SRI). For Ising systems with SRI, the predictions obtained by renormalization group calculation [1, 8] have been successfully checked by the short-time Monte Carlo simulation [4, 9]. In recent years there has been increasing interest in the investigation of the critical properties of systems with long-range interaction (LRI) decaying at large distances r by a power law as $r^{-d-\rho}$, with d the spatial dimension and ρ the parameter controlling the range of the interaction [10–13]. Their results agree with the well known fact that the critical equilibrium properties are modified by the presence of LRI [14–23]. Recently, how the short-time critical behaviour depends upon the interaction range, has been discussed in the kinetic spherical model [6] and the Ginzburg–Landau model [7]. Experimentally, systems with LRI could be found in ionic solutions where the Coulomb interaction is partially screened [24, 25]. The LRI is important in some low-dimensional systems such as the conjugated polymers [26, 27].

In this paper, we study the short-time critical behaviour of anisotropic cubic systems with long-range exchange interaction. These systems are more realistic in solid state materials. In equilibrium at temperature T the Hamiltonian describing these systems is given by

$$H[s] \equiv \int \mathrm{d}^d x \left\{ \frac{a}{2}(\nabla s)^2 + \frac{b}{2}(\nabla^{\sigma/2} s)^2 + \frac{\tau}{2} s^2 + \frac{g_i}{4!}(s^2)^2 + \frac{g_a}{4!} \sum_{\alpha=1}^{n} (s^\alpha)^4 \right\} \qquad (1)$$

where $s = (s^\alpha)$ are n-component order parameter fields, and $s^2 = \sum_{\alpha=1}^{n} s^\alpha s^\alpha$; τ is proportional to the reduced temperature $T/T_c - 1$; g_i and g_a are the coupling constants for the isotropy and the anisotropy, respectively. The SRI model corresponds to $a = 1$ and $b = 0$ for $\rho \geqslant 2$, whereas for the pure LRI model $\sigma = \rho < 2$, $a = 0$ and $b = 1$. The cubic g_a-term in equation (1) suggests that the spin interaction reacts to the lattice structure (crystalline anisotropy) in cubic crystals.

Since the case $0 < \sigma < d/2$ is covered by a mean-field-theoretic description, and since for $\sigma > 2$ and $d > 2$ the model (1) belongs to the same universality class as the SRI model, we will restrict ourselves in the present paper to the range $d/2 < \sigma < \min(2, d)$.

The static and equilibrium dynamic scaling properties of anisotropic cubic models without LRI have been the subject of a large number of studies, and are now well understood [28–34]. The pure LRI case has also been investigated in [39]. The cubic g_a-term appears in the Hamiltonian of the systems, as single-ion interactions, which is important at structure phase transitions. These models exhibit different types of continuous and first-order phase transitions, depending upon n, d, g_a and also σ.

The dynamics to be discussed here, which is called the model A dynamics [40], is controlled by the Langevin equation

$$\partial_t s^\alpha(x, t) = -\lambda \frac{\delta H[s]}{\delta s^\alpha(x, t)} + \xi^\alpha(x, t)$$

where λ is the kinetic coefficient. The random forces $\xi = (\xi^\alpha)$ are assumed to be Gaussian distributed

$$\langle \xi^\alpha(x, t) \rangle = 0 \qquad \langle \xi^\alpha(x, t) \xi^\beta(x', t') \rangle = 2\lambda \delta^{\alpha\beta} \delta(x - x') \delta(t - t').$$

As mentioned above, the initial non-equilibrium state is macroscopically prepared at some very high temperature $T_i \gg T_c$. The initial state $s_0(x) = s(x, 0)$ with short-range correlations corresponds to a Gaussian distribution $P[s_0] \propto \exp(-H^i[s_0])$ where

$$H^i[s_0] \equiv \int \mathrm{d}^d x \, \frac{\tau_0}{2} [s_0(x) - m_0(x)]^2$$

with τ_0 being proportional to $T_i/T_c - 1$ and $m_0(x)$ the (spatially varying) initial order parameter. By a naive dimensional analysis, $\tau_0 \sim \mu^\sigma$ (where μ is a renormalization momentum scale), the physically interesting fixed point is $\tau_0^* = +\infty$, which corresponds to a sharply initial non-equilibrium state with initial order m_0 and zero correlation length.

Introducing a Martin–Siggia–Rose response field $\tilde{s}(x,t)$ [41, 42], the generating functional for all non-equilibrium connected correlation and response functions is given by

$$W[h,\tilde{h}] = \ln \int \mathcal{D}(\mathrm{i}\tilde{s}, s) \exp\left\{-\mathcal{L}[\tilde{s}, s] - H^i[s_0] + \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x\, (hs + \tilde{h}\tilde{s})\right\} \tag{2}$$

where

$$\mathcal{L}[\tilde{s}, s] \equiv \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x \sum_{\alpha=1}^{n} \left\{ \tilde{s}^\alpha \left[\partial_t + \lambda \left(\tau - a\,\nabla^2 + b(-\nabla^2)^{\sigma/2}\right) + \frac{\lambda g_i}{6} s^2 + \frac{\lambda g_a}{6} (s^\alpha)^2 \right] s^\alpha - \lambda (\tilde{s}^\alpha)^2 \right\}. \tag{3}$$

Here we have used a pre-point discretization with respect to time so that the step function $\Theta(t=0) = 0$. Then the contribution (proportional to $\Theta(0)$) to $\mathcal{L}[\tilde{s}, s]$ arising from the functional determinant det $[\delta\xi(x,t)/\delta s(x,t)]$ vanishes [43].

The scaling regime of the LRI model is valid only for $\sigma < \sigma_s \equiv 2 - \eta_{sr}$, where η_{sr} is the Fisher exponent at the SRI fixed point [18]; whereas for $\sigma > \sigma_s$ the scaling behaviour is described by the SRI model. At the borderline value $\sigma = \sigma_s$ the two descriptions yield equal values for the critical exponents [18, 20, 21]. Since the short-range exchange interaction is irrelevant for $d/2 < \sigma < \sigma_s$, one can consider only the pure LRI.

The added cubic term, i.e. the g_a-term in equation (1), breaks explicitly the O(n) invariance of the model, but preserves the cubic symmetry. The model described by the Hamiltonian (1) has four fixed points corresponding to the case of SRI or LRI: the Gaussian one, the Ising one, the Heisenberg one and the cubic one [31, 39]. The Gaussian fixed point is always unstable, and so is the Ising fixed point. The stability properties of the Heisenberg and the cubic fixed points depend on n, d and σ. For $n < n_c$, the Heisenberg fixed point is stable and the cubic one is unstable. The cubic term in the Hamiltonian represents only a correction to scaling near the Heisenberg fixed point. While for $n > n_c$, the cubic anisotropy is relevant, and the Heisenberg fixed point becomes the unstable one. That is because the anisotropic cubic interaction grows and the behaviour of the cubic interaction dominates over the Heisenberg behaviour. At $n = n_c$ the two fixed points coincide, a crossover from the Heisenberg behaviour to the cubic behaviour takes place. For the SRI case, field-theoretic studies predict that the marginal value n_c holds $2.8 < n_c \leqslant 3$ [34–37], while Monte Carlo simulations suggest $n_c \approx 3$ by use of finite-size scaling techniques [38].

In the present paper, we are interested in the scaling affected by both LRI and the cubic anisotropy in the regime $\sigma < \sigma_s$. Through dimensional analysis, the upper critical dimension of the LRI model is $d_c = 2\sigma$. We apply the ϵ-expansion theory to the LRI model in this regime with $\epsilon \equiv 2\sigma - d$. The critical initial order increase appears in the LRI model for $1 \leqslant d < d_c$. The scaling behaviour of the critical initial slip is governed by the exponents θ and θ'. They are computed as functions of d, σ and n at the LRI cubic fixed point. Our results show that the anisotropic cubic interaction affects the short-time critical behaviours for $1 \leqslant d < d_c$ and $n > n_c$.

The paper is organized as follows. In section 2, the LRI model with $\sigma < \sigma_s$ is studied by the ϵ expansion method. The scaling behaviour of the order parameter, correlation and response functions, as well as the corresponding critical initial slip exponents, are obtained. Section 3 contains conclusions and discussions.

2. The scalings and exponents

Since the SRI is irrelevant for $\sigma < \sigma_s$, in this section we take $a = 0$ and $b = 1$ in (3). For $g_i = g_a = 0$, the generating functional (2) becomes the Gaussian model which serves as the free part of a perturbation series. One must take into account the initial condition, by imposing the following boundary conditions:

$$\tilde{s}(x, \infty) = 0 \qquad s_0(x) = m_0(x) + {\tau_0}^{-1}\tilde{s}(x, 0).$$

The free response function $G_p(t, t') = \langle s_p(t)\tilde{s}_{-p}(t')\rangle_G$ and the free correlation function $C_p(t, t') = \langle s_p(t)s_{-p}(t')\rangle_G$ are, respectively,

$$G_p(t, t') = \Theta(t - t')\exp[-\lambda(p^\sigma + \tau)(t - t')]$$
$$C_p(t, t') = C_p^{(e)}(t - t') + C_p^{(i)}(t, t')$$

with the equilibrium part $C_p^{(e)}(t - t')$ and the initial (non-equilibrium) part $C_p^{(i)}(t, t')$ defined by

$$C_p^{(e)}(t - t') \equiv \frac{1}{\tau + p^\sigma}\exp[-\lambda(p^\sigma + \tau)|t - t'|]$$
$$C_p^{(i)}(t, t') \equiv \left(\tau_0^{-1} - \frac{1}{\tau + p^\sigma}\right)\exp[-\lambda(p^\sigma + \tau)(t + t')].$$

One now sets a perturbation expansion ordered by the number of loops in the Feynman diagrams. It is convenient to consider the Dirichlet boundary conditions $\tau_0 = +\infty$ and $m_0(x) = 0$. The general case is recovered by treating the parameters ${\tau_0}^{-1}$ and $m_0(x)$ as additional perturbations. The model (2) with Dirichlet boundary conditions must be renormalized. For this purpose note that the free correlation function simplifies to

$$C_p^{(D)}(t, t') \equiv \frac{1}{\tau + p^\sigma}\left\{\exp[-\lambda(p^\sigma + \tau)|t - t'|] - \exp[-\lambda(p^\sigma + \tau)(t + t')]\right\}.$$

Through dimensional analysis, the critical dimension $d_c = 2\sigma$, and hence it is convenient to make an expansion in $\epsilon = 2\sigma - d$. A perturbation calculation of Green functions leads to integrals which are ultraviolet-divergent at d_c. We will apply the dimensional regularization with minimal subtraction scheme [44] to render these integrals finite, and introduce renormalized quantities through multiplicative factors

$$\begin{aligned} &s_b = Z_s^{1/2}s \qquad \tilde{s}_b = Z_{\tilde{s}}^{1/2}\tilde{s} \qquad \lambda_b = (Z_s/Z_{\tilde{s}})^{1/2}\lambda \\ &g_{ib} = K_d^{-1}\mu^\epsilon Z_s^{-2}Z_{u_i}u_i \qquad g_{ab} = K_d^{-1}\mu^\epsilon Z_s^{-2}Z_{u_a}u_a \\ &\tau_b = Z_s^{-1}Z_\tau\tau \qquad \tau_{0b} = (Z_{\tilde{s}}/Z_s)^{1/2}\tau_0 \qquad \tilde{s}_{0b} = (Z_{\tilde{s}}Z_0)^{1/2}\tilde{s}_0 \end{aligned} \tag{4}$$

where the subscript b denotes the bare quantity and $K_d \equiv 2^{1-d}\pi^{-d/2}[\Gamma(d/2)]^{-1}$.

Since the equilibrium part of the Dirichlet correlator $C_p^{(e)}(t - t')$ gives the same ultraviolet divergences as those of the equilibrium theory at $t = t'$, the renormalization constants Z_s, $Z_{\tilde{s}}$, Z_τ, Z_{u_i} and Z_{u_a} can cure these divergences. On the other hand, the non-equilibrium initial conditions break the translational invariant with respect to time. That is to say that there are divergences arising from the initial part $C_p^{(i)}(t, t')$ at $t + t' = 0$, hence one encounters a new renormalization constant Z_0. A naive power counting and the Ward identities:

$$s_0(x) = 0 \qquad \dot{s}_0(x) = 2\lambda\tilde{s}_0(x)$$

reveal that this new renormalization is required only in a two-point response function $\langle s(x, t)\tilde{s}(x', 0)\rangle$.

At a fixed value of σ, a two-loop calculation gives the following renormalization constants which render the equilibrium Green functions finite:

$$Z_s = 1 \tag{5}$$

$$Z_{\tilde{s}} = 1 - \frac{n+2}{6\epsilon} B_\sigma u_i^2 - \frac{1}{\epsilon} B_\sigma u_i u_a - \frac{1}{2\epsilon} B_\sigma u_a^2 \tag{6}$$

$$Z_{u_i} = 1 + \frac{n+8}{6\epsilon} u_i + \frac{1}{\epsilon} u_a + \left[\frac{(n+8)^2}{36\epsilon^2} - \frac{5n+22}{36\epsilon} D_\sigma \right] u_i^2 + \left(\frac{n+12}{4\epsilon^2} - \frac{1}{\epsilon} D_\sigma \right) u_i u_a + \left(\frac{5}{4\epsilon^2} - \frac{1}{4\epsilon} D_\sigma \right) u_a^2 \tag{7}$$

$$Z_{u_a} = 1 + \frac{2}{\epsilon} u_i + \frac{3}{2\epsilon} u_a + \left(\frac{n+20}{6\epsilon^2} - \frac{n+14}{12\epsilon} D_\sigma \right) u_i^2 + \left(\frac{11}{2\epsilon^2} - \frac{2}{\epsilon} D_\sigma \right) u_i u_a + \left(\frac{9}{4\epsilon^2} - \frac{3}{4\epsilon} D_\sigma \right) u_a^2 \tag{8}$$

$$Z_\tau = 1 + \frac{n+2}{6\epsilon} u_i + \frac{1}{2\epsilon} u_a + \left[\frac{(n+2)(n+5)}{36\epsilon^2} - \frac{n+2}{24\epsilon} D_\sigma \right] u_i^2 + \left(\frac{n+5}{6\epsilon^2} - \frac{1}{4\epsilon} D_\sigma \right) u_i u_a + \left(\frac{1}{2\epsilon^2} - \frac{1}{8\epsilon} D_\sigma \right) u_a^2 . \tag{9}$$

Here we have introduced

$$B_\sigma \equiv K_{2\sigma}^{-1} \int \frac{\mathrm{d}^{2\sigma} x}{(2\pi)^{2\sigma}} [1 + x^\sigma + (e + x)^\sigma]^{-2} x^{-\sigma}$$

with $\boldsymbol{e}$ a unit vector in the 2σ-dimensional space, and

$$D_\sigma \equiv \psi(1) - 2\psi(\sigma/2) + \psi(\sigma)$$

with $\psi(x)$ being the logarithmic derivative of the gamma function. For the particular case $\sigma = 2$, one has $B_2 = \frac{1}{2} \ln \frac{4}{3}$, and $D_2 = 1$.

In order to determine the renormalization constant Z_0, we calculate the two-point function $\langle s(-q, t)\tilde{s}(q, t')\rangle$, with one leg attached to the initial surface $t' = 0$

$$\langle s(-q, t)\tilde{s}(q, 0)\rangle = \int_0^\infty \mathrm{d}t' \, \langle s(-q, t)\tilde{s}(q, t')\rangle^{(e)} \Gamma_{10}^{(i)}(q, t')$$

by using the graphs of figure 1. The factor $\langle s(-q, t)\tilde{s}(q, t')\rangle^{(e)}$ denotes the contribution to the two-point function coming only from the equilibrium part $C_p^{(e)}(t, t')$, whereas the residual factor $\Gamma_{10}^{(i)}(q, t')$ is the sum of the amplitudes with at least one initial part $C_p^{(i)}(t, t')$. In these diagrams $C_p^{(D)}(t, t')$ and $G_p(t, t')$ are represented by full lines without and with arrows, respectively. The small circle means that one time argument is set equal to zero. Since the diagrams containing the vertices g_s or g_c are similar, the vertices are not shown in figure 1 explicitly.

We write the singular part of $\Gamma_{10}^{(i)}$ at the critical point $\tau = 0$ in the form

$$\Gamma_{10}^{(i)}(q = 0, t) = I_1 - \lambda \left(\frac{n+2}{6} g_i + \frac{1}{2} g_a \right) I_2 + \lambda^2 \left[\left(\frac{n+2}{6} \right)^2 g_i^2 + \frac{n+2}{6} g_i g_a + \frac{1}{4} g_a^2 \right] (2I_3 + I_5) + \lambda^2 \left(\frac{n+2}{6} g_i^2 + g_i g_a + \frac{1}{2} g_a^2 \right) I_4 \tag{10}$$

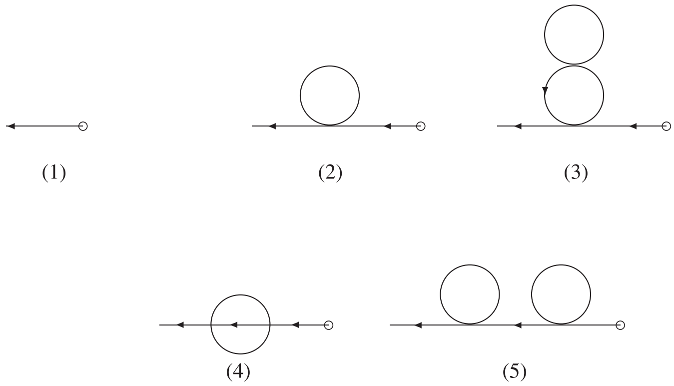

Figure 1. Diagrams contributing to $\Gamma_{10}^{(i)}(q,t)$ up to two-loops.

where I_j with $j = 1, 2, 3, 4, 5$ is the contribution of the jth diagram in figure 1. These contributions up to two-loop order are given by

$$I_1 = \delta(t)$$

$$I_2 = \int \frac{\mathrm{d}^d p}{(2\pi)^d} C_p^{(i)}(t,t) = -\frac{1}{\sigma} K_d \Gamma\left(1 - \frac{\epsilon}{\sigma}\right)(2\lambda t)^{-1+\epsilon/\sigma}$$

$$I_3 = \int_0^t \mathrm{d}t' \int \frac{\mathrm{d}^d p}{(2\pi)^d} C_p^{(i)}(t,t) \int \frac{\mathrm{d}^d p'}{(2\pi)^d} G_{p'}(t,t') C_{p'}^{(D)}(t,t')$$
$$= -K_d^2 \frac{\Gamma^2(1-\epsilon/\sigma)}{\sigma\lambda\epsilon}\left[\frac{\Gamma^2(1+\epsilon/\sigma)}{\Gamma(1+2\epsilon/\sigma)} - \frac{1}{2}\right](2\lambda t)^{-1+2\epsilon/\sigma}$$

$$I_4 = \int_0^t \mathrm{d}t' \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{\mathrm{d}^d p'}{(2\pi)^d} G_{p+p'}(t,t')\big(2C_p^{(i)}(t,t')C_{p'}^{(e)}(t,t') + C_p^{(i)}(t,t')C_{p'}^{(i)}(t,t')\big)$$
$$= \frac{1}{\sigma\lambda} K_d^2 \Gamma(1 - \frac{2\epsilon}{\sigma})\left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma - \frac{1}{\epsilon} + \mathrm{O}(\epsilon)\right)(2\lambda t)^{-1+2\epsilon/\sigma}$$

$$I_5 = \int_0^t \mathrm{d}t' \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{\mathrm{d}^d p'}{(2\pi)^d} C_p^{(i)}(t,t) C_{p'}^{(i)}(t',t') = \frac{1}{2\sigma\lambda\epsilon} K_d^2 \Gamma^2\left(1 - \frac{\epsilon}{\sigma}\right)(2\lambda t)^{-1+2\epsilon/\sigma}.$$

We renormalize now according to (4)–(8) the (bare) quantities entering this expression (10). The residual singularity is then removed by requiring [1]

$$Z_0^{-1/2} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}\omega t} \Gamma_{10}^{(i)}(q=0,t)_b = \text{finite for } \epsilon \to 0.$$

Here the subscript b denotes the expression of $\Gamma_{10}^{(i)}$ obtained above in which only bare quantities appear. From this condition we compute Z_0 as

$$Z_0 = 1 + \frac{n+2}{6\epsilon} u_i + \frac{1}{2\epsilon} u_a + \frac{n+2}{12\epsilon^2}\left[\frac{n+5}{3} + \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right)\epsilon\right] u_i^2$$
$$+ \frac{1}{2\epsilon^2}\left[\frac{n+5}{3} + \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right)\epsilon\right] u_i u_a$$
$$+ \frac{1}{4\epsilon^2}\left[2 + \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right)\epsilon\right] u_a^2. \quad (11)$$

Introducing the renormalized non-equilibrium connected Green function of N s-fields, $\tilde{N}$ $\tilde{s}$-fields, and M $\tilde{s}_0$-fields, i.e. $G^M_{N\tilde{N}} = \langle s^N \tilde{s}^{\tilde{N}} \tilde{s}_0^M \rangle$, then the renormalization group equation is expressed as

$$[\mu\partial_\mu + \zeta\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta_i\partial_{u_i} + \beta_a\partial_{u_a} + \zeta\tau_0^{-1}\partial_{\tau_0^{-1}} + \tfrac{1}{2}(N\gamma + \tilde{N}\tilde{\gamma} + M(\tilde{\gamma} + \gamma_0))]G^M_{N\tilde{N}} = 0. \qquad (12)$$

The Wilson functions entering the renormalization group equations are defined by

$$\gamma \equiv \mu\partial_\mu \ln Z_s|_0 \qquad \beta_i \equiv \mu\partial_\mu u_i|_0 \qquad \beta_a \equiv \mu\partial_\mu u_a|_0 \qquad \tilde{\gamma} \equiv \mu\partial_\mu \ln Z_{\tilde{s}}|_0$$

$$\kappa \equiv \mu\partial_\mu \ln \tau|_0 \qquad \zeta \equiv \mu\partial_\mu \ln \lambda|_0 = \tfrac{1}{2}(\tilde{\gamma} - \gamma) \qquad \gamma_0 \equiv \mu\partial_\mu \ln Z_0|_0$$

and are computed perturbatively from equations (5)–(9). The symbol $|_0$ means that μ-derivatives are calculated at fixed bare parameters. At the two-loop level, the Wilson functions, which agree with those of the equilibrium critical dynamics, are given by

$$\beta_i = -\epsilon u_i + \frac{n+8}{6}u_i^2 + u_a u_i - \frac{5n+22}{18}D_\sigma u_i^3 - 2D_\sigma u_a u_i^2 - \frac{1}{2}D_\sigma u_a^2 u_i$$

$$\beta_a = -\epsilon u_a + 2u_a u_i + \frac{3}{2}u_a^2 - \frac{n+14}{6}D_\sigma u_a u_i^2 - 4D_\sigma u_a^2 u_i - \frac{3}{2}D_\sigma u_i^3$$

$$\gamma_s = 0$$

$$\gamma_{\tilde{s}} = \frac{n+2}{3}B_\sigma u_i^2 + 2B_\sigma u_a u_i + B_\sigma u_a^2$$

$$\zeta = \frac{n+2}{6}B_\sigma u_i^2 + B_\sigma u_a u_i + \frac{1}{2}B_\sigma u_a^2$$

$$\kappa = \frac{n+2}{6}u_i + \frac{1}{2}u_a - \frac{n+2}{12}D_\sigma u_i^2 - \frac{D_\sigma}{2}u_a u_i - \frac{D_\sigma}{4}u_a^2.$$

The new Wilson function γ_0, which is related to the initial order parameter, is given by

$$\gamma_0 = -\frac{n+2}{6}u_i - \frac{1}{2}u_a - \left(\frac{n+2}{6}u_i^2 + u_a u_i + \frac{1}{2}u_a^2\right)\left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right). \qquad (13)$$

By solving algebraically the equations $\beta_i(u_i, u_a) = 0$ and $\beta_i(u_i, u_a) = 0$, one finds four fixed points: Gaussian, Ising, Heisenberg and cubic. Their stability has been discussed in [39]. How the former three fixed points govern the short-time behaviour has also been investigated in the isotropic systems [7]. Here we are only interested in the infrared LRI cubic fixed point

$$u_i^* = \frac{2\epsilon}{n}\left[1 + \frac{2(n-1)(6-n)}{3n^2}D_\sigma\epsilon\right] + \mathrm{O}(\epsilon^3)$$
$$u_a^* = \frac{2\epsilon}{3n}\left[n - 4 + \frac{2(n-1)(n^2+6n-24)}{3n^2}D_\sigma\epsilon\right] + \mathrm{O}(\epsilon^3) \qquad (14)$$

and subsequently the values of the Wilson functions at this point. The eigenvalues λ_i and λ_a corresponding to u_i^* and u_a^*, respectively, are

$$\lambda_i = \epsilon - \frac{2(n-1)(n^2+12)}{3n^2(n+2)}D_\sigma\epsilon^2 + \mathrm{O}(\epsilon^3)$$
$$\lambda_a = \frac{(n-4)\epsilon}{3n}\left[1 - \frac{2(n-1)(n^3-4n^2-36n+48)}{3n^2(n-4)(n+2)}D_\sigma\epsilon\right] + \mathrm{O}(\epsilon^3). \qquad (15)$$

The LRI cubic fixed point is stable for $\lambda_i > 0$ and $\lambda_a > 0$. From $\lambda_i(n_c) = 0$ or $u_a^*(n_c) = 0$, one finds

$$n_c = 4 - 2D_\sigma\epsilon + \mathrm{O}(\epsilon^2) \tag{16}$$

where a crossover phenomenon between the LRI Heisenberg fixed point and the LRI cubic one takes place. For $n > n_c$ the critical scaling behaviour is governed by the LRI cubic fixed point. For fixed d (or σ), n_c decreases as σ increases (or d decreases).

Using dimensional analysis and the solution of equation (12), we find that the connected Green function at the fixed point $w^* = (u_i^*, u_a^*)$ displays the scaling behaviour:

$$G^M_{N\tilde{N}}(\{x,t\}, \tau, \tau_0^{-1}, \lambda, u_i^*, u_a^*, \mu) = l^{\frac{1}{2}(d-\sigma+\eta_s)N+\frac{1}{2}(d+\sigma+\eta_{\tilde{s}})\tilde{N}+\frac{1}{2}(d+\sigma+\eta_{\tilde{s}}+\eta_0)M}$$
$$\times\, G^M_{N\tilde{N}}\left(\left\{lx, l^{\sigma+\zeta(u^*)}t\right\}, \tau l^{-\sigma+\kappa(u^*)}, \tau_0^{-1} l^{\sigma+\zeta(u^*)}, \lambda, u_i^*, u_a^*, \mu\right) \tag{17}$$

where $\eta_s \equiv \gamma(w^*)$, $\eta_{\tilde{s}} \equiv \tilde{\gamma}(w^*)$, and $\eta_0 \equiv \gamma_0(w^*)$ are the anomalous dimensions of s, $\tilde{s}$ and $\tilde{s}_0$,

In order to identify the critical exponents one can compare the standard scaling form of the two-point correlation function

$$G^0_{20}(x-x', t, t', \tau) = |x-x'|^{-(d-2+\eta)} f\left(\frac{|x-x'|}{\xi}, \frac{|x-x'|}{t^{1/z}}, \frac{|x-x'|}{t'^{1/z}}\right)$$

to equation (15) in which we have set $N = 2$, $\tilde{N} = M = 0$ and $lx = 1$. Here $\xi \equiv \tau^{-\nu}$.

In this way we find the long-time critical exponents of the anisotropic cubic systems. They can be calculated by the relations $\eta \equiv 2 - \sigma + \eta_s$, $z \equiv \sigma + \zeta(w^*)$, and $1/\nu \equiv \sigma - \kappa(w^*)$. To second order in ϵ they are, respectively,

$$\eta = 2 - \sigma \tag{18}$$

$$\nu = \frac{1}{\sigma} + \frac{2(n-1)}{3\sigma^2 n}\left\{1 + \left[\frac{2(n-1)}{3\sigma n} - \frac{n^2 - 18n + 24}{3n^2}D_\sigma\right]\epsilon\right\}\epsilon \tag{19}$$

$$z = \sigma + \frac{2(n-1)(n+2)}{9n^2}B_\sigma\epsilon^2 \tag{20}$$

which agree with the results of [39].

The short-time scaling behaviour of correlation and response functions can be obtained by a short-time expansion of the fields $s(x,t)$ and $\tilde{s}(x,t)$, as done in [1]. By means of Green functions (15), one will find for $t \to 0$

$$s(x,t) = t^{1+\eta_0/(2z)}\phi(t/\xi^z)\tilde{s}_0(x) + \cdots \tag{21}$$

$$\tilde{s}(x,t) = t^{\eta_0/(2z)}\tilde{\phi}(t/\xi^z)\tilde{s}_0(x) + \cdots \tag{22}$$

where $\phi(0)$ and $\tilde{\phi}(0)$ are finite quantities. Combining them with equation (15), one can derive the following behaviour of the full response and correlation functions for $t > 0$ but $t' \to 0$:

$$G(p,t,t') = p^{-2+\eta+z}\left(\frac{t}{t'}\right)^{\theta} f_G\left(p\xi, p^z t\right) \tag{23}$$

$$C(p,t,t') = p^{-2+\eta}\left(\frac{t}{t'}\right)^{\theta-1} f_C\left(p\xi, p^z t\right) \tag{24}$$

with the scaling function f_G and f_C. Here we defined the initial slip exponent $\theta \equiv -\eta_0/(2z)$ and computed it to second order in ϵ

$$\theta = \frac{\epsilon(n-1)}{3\sigma n}\left\{1 - \left[\frac{n^2 - 18n + 24}{6n^2}D_\sigma - \frac{2(n+2)}{3\sigma n}\ln 2\right]\epsilon\right\}. \tag{25}$$

Let us now discuss the scaling form of the order parameter which relaxes from a non-zero initial value m_0 to zero. In this case, $m_0(x)$ is considered as a time-independent source coupled to $\tilde{s}_0(x)$ [1]. Taking a homogeneous source $m_0(x) = m_0$, but keeping still $\tau_0^* = +\infty$, the power law of the time-dependent order parameter $m(t) \equiv \langle s(x,t)\rangle|_{\tilde{h}=h=0}$ is given by

$$m(t) = m_0 t^{\theta'} f_m\left(m_0 t^{\theta' + (d-2+\eta)/2z}, \tau t^{1/\nu z}\right) \tag{26}$$

where the scaling function $f_m(0, 0)$ is finite, and $f_m(x, 0) \sim 1/x$ for $x \to \infty$. In equation (26), the exponent θ' is defined by $\theta' \equiv -(\eta_s + \eta_{\tilde{s}} + \eta_0)/(2z)$. To second order in ϵ it has the value

$$\theta' = \frac{\epsilon(n-1)}{3\sigma n}\left\{1 - \left[\frac{n^2 - 18n + 24}{6n^2} D_\sigma - \frac{2(n+2)}{3\sigma n}(\ln 2 - \sigma B_\sigma)\right]\epsilon\right\}. \tag{27}$$

From equations (25) and (27), it is easily verified that for $n = 1$ and 2, the short-time exponents θ and θ' are the Gaussian-like and Ising-like, respectively. That is because the cubic fixed point is degenerate with the Gaussian fixed point for $n = 1$ and with the Ising fixed point for $n = 2$ [28, 39]. To second order in ϵ, they meet the corresponding Heisenberg exponents [7] at $n = n_c$.

3. Discussions and conclusions

It is believed that the singularity of the temporal correlation is essential to the short-time scaling and the scaling can emerge in the early stage of the evolution even though all spatial correlations are still short-ranged.

The system with LRI is now rapidly quenched to a temperature $T \simeq T_c$. The order parameter will undergo a relaxation process displaying an initial increase. As long as the correlations are short-ranged and the spatial dimension d is smaller than the critical dimension $d_c = 2\sigma$, the order parameter follows a mean-field ordering process because the mean-field critical temperature $T_c^{(\mathrm{mf})}$ is larger than the actual critical temperature T_c. This ordering causes an amplification of the initial order parameter. For $d > d_c$ mean-field theory applies and there is no critical increase.

When the system with LRI has a cubic anisotropy, one has to consider the influence of the cubic term in the Hamiltonian on the critical behaviour. The importance of the cubic term is due to the fact that in a real crystal the crystalline structure gives rise to anisotropies which are mainly of cubic type [38]. Thus real crystals are better described by mixed actions in which both the isotropic and the cubic term are presented. Two-loop calculation shows that at

$$n = n_c = 4 - 2D_\sigma\epsilon$$

the scaling behaviours governed by the LRI Heisenberg fixed point crosses over to those by the LRI cubic one. For $n < n_c$, the Heisenberg fixed point is stable and the cubic one is unstable. The cubic term in the Hamiltonian represents only a correction to scaling near the Heisenberg fixed point, which is similar to the SRI case [31]. While for $n > n_c$, the cubic anisotropy is relevant, and the Heisenberg fixed point becomes the unstable one. That is because the anisotropic cubic interaction grows and the behaviour of the cubic interaction dominates over the Heisenberg behaviour. Since the cubic fixed point is degenerate with the Gaussian fixed point for $n = 1$ and with the Ising fixed point for $n = 2$, the cubic behaviour is reduced as are those of the Gaussian and Ising systems, respectively.

In the following we focus on the pole the cubic anisotropy plays in the short-time scaling behaviour. Let us first note that both the response and the correlation functions measure the fluctuations of the order parameter fields. For the short time after quench the cubic scaling behaviours are governed by the initial slip exponents θ and θ'. Since the initial slip exponents θ and θ' are positive, one expects, according to equations (23)–(27), an initial increase of the fluctuations. The stronger the fluctuations, the greater the values of θ and θ'. Of course, the increase depends upon σ, d and n. Since fluctuations are reduced as the dimension becomes larger or interactions of longer range (σ smaller), θ and θ' decrease when d increases or σ decreases. But the relationship between n and the initial increase seems delicate. For fixed σ and d, more internal degrees of freedom (larger n) help the fluctuations increase. But for large σ, the fluctuations decrease when n exceeds some threshold value, which is probably because the interactions among a huge number of internal degrees strengthen the mean fields, but suppress the fluctuations.

Our results can also be compared with those of the Heisenberg systems with LRI [7]. The infrared Heisenberg fixed point to order ϵ^2 is located at

$$u_H^* = \frac{6\epsilon}{n+8}\left[1 + \frac{2(5n+22)}{(n+8)^2} D_\sigma \epsilon\right] + \mathrm{O}(\epsilon^3). \tag{28}$$

Here the subscript H means the Heisenberg systems. The critical initial slip exponent θ' in this system can also be computed to second order in ϵ [7]:

$$\theta'_H = \frac{\epsilon(n+2)}{2\sigma(n+8)}\left\{1 + \left[\frac{7n+20}{(n+8)^2} D_\sigma + \frac{12(\ln 2 - \sigma B_\sigma)}{\sigma(n+8)}\right]\epsilon\right\}. \tag{29}$$

In cubic systems the behaviours that the initial exponents are dependent on d, σ and n, are similar to those in the Heisenberg systems. As mentioned before, for $n > n_c$, since the cubic behaviour overcomes the Heisenberg behaviour, one will expect that the cubic exponent $\theta' > \theta'_H$. But, in fact, it is not so simple. Due to the competition between the isotropic interaction and the anisotropic cubic interaction, the different poles in the fluctuations they play (because the initial increase of the fluctuations are decided by the anisotropic interaction g_a and the isotropic interactions g_i), there exist three distinct regimes in the cubic systems for fixed d and σ. For $1 \leqslant n < n_c < 4$, the isotropic interaction helps to increase the fluctuations, while the anisotropic interaction grows, but it suppresses the fluctuations (see the ϵ-term in the second equation u_a^* in equation (14)). Hence $\theta' < \theta'_H$ in this regime. At $n = n_c$, since to order ϵ^2 the affect of the anisotropy disappears, all the fluctuations are equal in both the cubic systems and the Heisenberg systems, so $\theta' = \theta'_H$. When $n > n_c$, the behaviour of the cubic interaction dominates over the Heisenberg behaviour so that $\theta' > \theta'_H$. Due to the effect of the mean fields, the fluctuations decrease when n exceeds some threshold value in both systems. In addition, the D_σ-term in equation (27) changes to become negative for some large n in the cubic systems. Hence $\theta' < \theta'_H$ for large n. One may easily check this conclusion by computing θ' and θ'_H for $n = 1, 2, 3, n_c$ and ∞ in three dimensions. This conclusion also holds for the initial slip exponent θ.

At $\sigma = \sigma_s \equiv 2 - \eta_{sr}$ (where $\eta_{sr} = [(n-1)(n+2)/54n^2]\epsilon'^2 + \mathrm{O}(\epsilon'^3)$ and $\epsilon' \equiv 4 - d$ [31]) and fixed d, our results recover the SRI results with the cubic anisotropy [45]. That is to say that at $\sigma = \sigma_s$ the LRI scaling behaviours cross over to the SRI ones. For $\sigma > \sigma_s$, the scaling regime is governed by the SRI fixed points. The LRI scaling regime is only valid for $\sigma < \sigma_s$.

We summarize now our results. We studied the short-time critical behaviour of the anisotropic cubic systems with LRI in the ϵ-expansion up to two-loop order. We observed an initial critical increase for dimensions smaller than d_c and for the interaction range $d/2 < \sigma < d$. We obtained the universal critical exponents θ and θ' of the initial slip as functions of d, n, and the interaction range parameter σ. Our results show that the anisotropic cubic interaction affects the short-time critical dynamics.

Finally, we would like to mention that a check of our results by Monte Carlo simulations will soon be available [46]. They are starting Monte Carlo simulation of 1D LRI Ising model, and the preliminary results are encouraging. One of their results, which is obtained from the simulation of lattice size $L = 4000$ and near the critical point, shows that $\theta' = 0.164\,77$ for $m_0 = 0.01$. This value agrees with our theoretical value $\theta' = 0.167\,34$ for the 1D anisotropic cubic XY system ($n = 2$) and $\sigma = 0.7$. The experiments which try to test our results may be carried out in physical systems such as $LaAlO_3$, $SrTiO_3$ and $KMnF_3$ (where structural phase transitions have been proposed to be in the universality class of the cubic anisotropic model with $d = n = 3$ [33, 47]), or ionic solutions and the conjugated polymers with LRI.

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